Examiners' Report: Final Honour School of Mathematics Part C Trinity Term 2020

March 9, 2021

Part I

A. STATISTICS

- **Numbers and percentages in each class.**

  See Table 1, page 1.

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted for the FHS of Mathematics Part C.

- **Marking of scripts.**
  The dissertations and mini-projects were double marked. The remaining scripts were all single marked according to a pre-agreed marking scheme which was very closely adhered to. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**
  See Table 5 on page 8.

Table 1: Numbers in each class

	Number					Percentages %				
	2020	(2019)	(2018)	(2017)	(2016)	2020	(2019)	(2018)	(2017)	(2016)
I	63	(58)	(53)	(48)	(44)	67.74	(57.43)	(56.99)	(57.14)	(50.57)
II.1	30	(40)	(26)	(23)	(31)	32.26	(39.6)	(27.96)	(27.38)	(35.63)
II.2	0	(2)	(13)	(12)	(9)	0	(1.98)	(13.98)	(14.29)	(10.34)
III	0	(1)	(1)	(1)	(3)	0	(0.99)	(1.08)	(1.19)	(3.45)
F	0	(0)	(0)	(0)	(0)	0	(0)	(0)	(0)	(0)
Total	93	(101)	(93)	(84)	(87)	100	(100)	(100)	(100)	(100)

B. New examining methods and procedure in the 2020 examinations

In light of Covid 19, the department took steps to mitigate the impact of the pandemic on academic performance. This included changing the examinations to open-book version of the standard exam papers, reducing the units required from 8 to 6, the introduction of the safety net and Declared to have Deserved Masters. In addition, the method of assessing mitigating circumstances at the exam board was changed. An additional hour was also added on to the Mathematics exam duration to allow candidates the technical time to download and submit their examination papers via Weblearn. Given the unusual circumstances and impact of Covid-19, ranking was only used for the purposes of awarding prizes. The introduction of the safety net (which was applied to cohorts) meant that the overall average and hence rank was not well defined.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

Due to the uncertainty with the pandemic, the department decided that exams will be taken online for Trinity Term 2021.

D. Notice of examination conventions for candidates

The first notice to candidates was issued on 19th February 2020 and the second notice on 6th May 2020. These contain details of the examinations and assessments.

All notices and the examination conventions for 2020 examinations are on-line at http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments.

Part II

A. General Comments on the Examination

The examiners would like to convey their grateful thanks for their help and cooperation to all those who assisted with this year's examination, either as assessors or in an administrative capacity. The chairman would particularly like to thank Nicole Collins for administering the whole process with efficiency, and also to thank Elle Styler, Charlotte Turner-Smith and Waldemar Schlackow. In addition the internal examiners would like to express their gratitude to Professor Richard Jozsa and Dr Jonathan Woolf for carrying out their duties as external examiners in a constructive and supportive way during the year, and for their valuable input at the final examiners' meetings.

Timetable

The examinations began on Monday 1st June and finished on Thursday 18th June.

Mitigating Circumstances Notice to Examiners and other special circumstances

In light of Covid 19, there was no separate panel meeting to discuss the individual notices to examiners. Even though the Mitigating Circumstances were initially reviewed at the preliminary meeting, all decisions on the outcome of these notices were decided at the final meeting alongside any cohort-wide decisions and the safety net being applied. The full board of examiners considered 10 notices in the final meeting. All candidates with certain conditions (such as dyslexia, dyspraxia, etc.) were given special consideration in the conditions and/or time allowed for their papers, as agreed by the Proctors. Each such paper was clearly labelled to assist the assessors and examiners in awarding fair marks.

Setting and checking of papers and marks processing

Following established practice, the questions for each paper were initially set by the course lecturer, with the lecturer of a related course involved as checker before the first draft of the questions was presented to the examiners. The course lecturers also acted as assessors, marking the questions on their course(s).

The internal examiners met in early January to consider the questions on Michaelmas Term courses, and changes and corrections were agreed with the lecturers where necessary. The revised questions were then sent to the external examiners. Feedback from external examiners was given to examiners, and to the relevant assessor for each paper for a response. The internal examiners met a second time late in Hilary Term to consider the external examiners' comments and assessor responses (and also Michaelmas Term course papers submitted late). The cycle was repeated for the Hilary Term courses, with two examiners' meetings in the Easter Vacation; the schedule here was much tighter.

Due to the Pandemic, Exam Papers were revised and set to be open book. Camera ready copy of each paper was signed off by the assessor, and then submitted to the Examination Schools.

Candidates accessed and downloaded their exam papers via the Weblearn system at the designated exam time. Exam responses were uploaded to Weblearn and made available to the Exam Board Administrator 48 hours after the exam paper had finished.

The process for Marking, marks processing and checking was adjusted accordingly to fit in with the online exam responses. Assessors had a short time period to return the marks on the mark sheets provided. A check-sum was also carried out to ensure that marks entered into the database were correctly read and transposed from the mark sheets.

All scripts and completed mark sheets were returned, if not by the agreed due dates, then at least in time for the script-checking process.

A team of graduate checkers, under the supervision of Nicole Collins and Elle Styler, reviewed the mark sheets for each paper of this examination, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way a number of errors were corrected, each change was approved by one of the examiners who were present throughout the process. A check-sum is also carried out to ensure that marks entered into the database are correctly read and transposed from the marks sheets.

Determination of University Standardised Marks

The Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average in each class over the last four years, together with recent historic data for Part C, the MPLS Divisional averages, and the distribution of classifications achieved by the same group of students at Part B.

The examiners followed established practice in determining the University standardised marks (USMs) reported to candidates. This leads to classifications awarded at Part C broadly reflecting the overall distribution of classifications which had been achieved the previous year by the same students.

We outline the principles of the calibration method.

The Department's algorithm to assign USMs in Part C was used in the same way as last year for each unit assessed by means of a traditional written examination. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part B classification of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers N_1, N_2 and N_3 are first computed for each paper: N_1, N_2 and N_3 are, respectively, the number of candidates taking the paper who achieved in Part B overall average USMs in the ranges $[70, 100]$, $[60, 69]$ and $[0, 59]$, respectively.

The algorithm converts raw marks to USMs for each paper separately (in each case, the raw marks are initially out of 50, but are scaled to marks out of 100). For each paper, the algorithm sets up a map $R \to U$ (R = raw, U = USM) which is piecewise linear. The graph of the map consists of four line segments: by default these join the points $(100, 100)$, $P_1 = (C_1, 72)$, $P_2 = (C_2, 57)$, $P_3 = (C_3, 37)$, and $(0, 0)$. The values of C_1 and C_2 are set by the requirement that the proportion of I and II.1 candidates in Part B, as given by N_1 and N_2, is the same as the I and II.1 proportion of USMs achieved on the paper. The value of C_3 is set by the requirement that P_2P_3 continued would intersect the U axis at $U_0 = 10$. Here the default choice of *corners* is given by U-values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs. The examiners have scope to make changes, usually by adjusting the position of the corner points P_1, P_2, P_3 by hand, so as to alter the map raw $\to$ USM, to remedy any perceived unfairness introduced by the algorithm, in particular in cases where the number of candidates is small. They also have the option to introduce additional corners.

Table 2 on page 6 gives the final positions of the corners of the piecewise linear maps used to determine USMs from raw marks. For each paper, P_1, P_2, P_3 are the (possibly adjusted) positions of the corners above, which together with the end points $(100, 100)$ and $(0, 0)$ determine the piecewise linear map raw $\to$ USM. The entries N_1, N_2, N_3 give the number of incoming firsts, II.1s, and II.2s and below respectively from Part B for that paper, which are used by the algorithm to determine the positions of P_1, P_2, P_3.

Following customary practice, a preliminary, non-plenary, meeting of examiners was held two days ahead of the plenary examiners' meeting to assess the results produced by the algorithm alongside the reports from assessors. The examiners reviewed each papers and report, considered whether open book examination process affected candidates and reviewed last year's stats. The examiners discussed the preliminary scaling maps and the preliminary class percentage figures. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low. These revised USM maps provided the starting point for a review of the scalings, paper by paper, by the full board of examiners.

Table 2: Position of corners of piecewise linear function

Paper	P_1	P_2	P_3	Additional corners	N_1	N_2	N_3
C1.1	15.28;37	26.6;57	38.6;72	50;100	6	3	0
C1.2	19.93; 37	35;60	50;100		4	4	0
C1.3	20.85; 37	36.3;57	50;100		8	10	0
C1.4	12.24;37	21.3;57	34.8;72	50;100	5	4	0
C2.1	16.43;37	28.6;57	40.6;72	50;100	4	4	0
C2.2	10.91;37	29;57	42;70	50;100	5	4	0
C2.4	13.79;37	30;57	36;70	40;80	3	4	0
C2.5	50;100	-	-	-	2	2	0
C2.6	50;100	-	-	-	3	0	0
C2.7	15.28;37	26.6;57	38.6;72	50;100	6	5	0
C3.1	17;37	33.9;62	44.4;77	50;100	6	1	0
C3.2	16.2;37	28.2;57	37.2;72	50;100	2	1	0
C3.3	9.59;37	33.2;72	50;100		3	2	0
C3.4	17.06;37	29.7;57	40;70	50;100	6	2	0
C3.5	50;100				4	1	0
C3.7	14.47;37	25.2;57	40;70	50;100	5	8	0
C3.8	5.74;37	20;50	40;72	50;100	7	10	0
C3.10	9;30	15;40	50;100		4	7	0
C4.1	10.63;37	24;60	30;72	50;100	7	1	0
C4.3	12.35;37	26;57	36;72	43;90	2	3	0
C4.6	22;55	32.2;72	50;100		4	0	1
C4.8	11.26;37	19.6;57	32;70	50;100	0	5	0
C5.1	16.03;37	27.9;57	38;70	50;100	0	4	0
C5.2	9.13;37	15;45	36;70	50;100	6	8	1
C5.5	15.57;37	27.1;57	40;67	50;100	4	14	1
C5.6	20;50	26.8;57	44;72	50;100	4	9	0
C5.7	14.30;37	24.9;57	35.4;72	50;100	5	3	1
C5.9	16.37;37	28.5;57	40;70	50;100	2	2	0
C5.11	13.33;37	23.2;57	44;70	50;100	4	12	1
C5.12	18.38;37	32;57	40;70	50;100	2	3	0
C6.1	18.33;37	30;57	42.4;72	50;100	3	7	1
C6.2	16.43;37	28.6;57	37;70	50;100	3	7	3
C6.3	50;100	-	-	-	2	2	0

Paper	P_1	P_2	P_3	Additional corners	N_1	N_2	N_3
C6.4	14.13;37	24.6;57	36.6;72	50;100	4	4	1
C7.4	14.48;37	25.2;57	42;70	50;100	2	2	0
C7.5	13.16;37	28;56	41;70	50;100	0	2	0
C7.6	13.04;37	19;50	23;60	50;100	0	1	0
C8.1	11.66;37	29;60	40;70	48;90	4	6	1
C8.2	16.95;37	29.5;57	37;72	50;100	4	4	1
C8.3	11.83;37	24;57	32.6;72	50;100	12	12	0
C8.4	13.73;37	26;55	35;70	50;100	11	9	1
C8.6	23.84;37	43;75	50;100		1	1	0
SC1	16.31;37	36;59	42;68	48;90	9	25	3
SC2	13.61;37	32;60	39;70	50;100	8	25	1
SC5	21.03;37	35;57	43;70	46;78	6	8	1
SC7	14.65;37	27;55	40;70	50;100	3	9	1
SC9	12.29;37	27;57	37;72	50;100	4	3	0
SC10	50;100				1	1	0

B. Equality and Diversity issues and breakdown of the results by gender

Table 4: Breakdown of results by gender

Class	Number								
	2020			2019			2018		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	16	47	63	8	50	58	6	47	53
II.1	4	26	30	9	31	40	7	19	26
II.2	0	0	0	0	2	2	3	10	13
III	0	0	0	0	1	1	1	0	1
F	0	0	0	0	0	0	0	0	0
Total	20	73	93	17	84	101	17	76	93

Class	Percentage								
	2020			2019			2018		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	80	64.38	72.19	47.06	59.52	57.43	35.29	61.84	56.99
II.1	20	35.62	27.81	52.94	36.9	39.6	41.18	25	27.96
II.2	0	0	0	0	2.38	1.98	17.65	13.16	13.98
III	0	0	0	0	1.19	0.99	5.88	0	1.08
F	0	0	0	0	0	0	0	0	0
Total	100	100	100	100	100	100	100	100	100

C. Detailed numbers on candidates' performance in each part of the exam

Data for papers with fewer than six candidates are not included.

Table 5: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
C1.1	9	37.11	4.73	71.33	7.33
C1.2	8	46.88	4.22	91.62	11.29
C1.3	18	45.94	3.61	87.22	11.46
C1.4	9	32.78	5.49	70.67	7.57
C2.1	8	38	5.37	70.75	9.6
C2.2	9	40.56	7.62	75	13.65
C2.4	7	33.86	4.67	66.14	10.04
C2.5	4	-	-	-	-
C2.6	3	-	-	-	-
C2.7	11	36.64	3.96	70.45	5.72
C3.1	7	44.57	4.5	82.14	11.45
C3.2	3	-	-	-	-
C3.3	5	-	-	-	-
C3.4	8	42.62	5.6	80.5	12.21
C3.5	5	-	-	-	-
C3.7	13	40.46	7.49	78.23	14.6
C3.8	17	33.65	10.59	68.24	16.79
C3.10	11	36.18	12.79	76.36	21.98
C4.1	8	30.62	8.86	70.75	14.16
C4.3	5	-	-	-	-
C4.6	5	-	-	-	-
C4.8	5	-	-	-	-
C5.1	4	-	-	-	-
C5.2	15	33.6	10.78	70.64	18.27
C5.5	18	38.28	5.73	69.5	11.41
C5.6	13	41.23	5.31	72.92	9.4
C5.7	9	35.56	7.78	73.78	13.25
C5.9	4	-	-	-	-
C5.11	17	38.76	6.65	68.88	8.52
C5.12	5	-	-	-	-
C6.1	6	37.33	4.84	65.67	6.02
C6.2	9	34.22	3.27	66.11	5.75
C6.3	4	-	-	-	-
C6.4	9	33.22	7.19	69	11.58

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
C7.4	4	-	-	-	-
C7.5	2	-	-	-	-
C7.6	1	-	-	-	-
C8.1	10	37.4	9.34	71.7	14.67
C8.2	8	35.12	6.6	68.88	12.79
C8.3	18	34.83	8.81	75.22	14.67
C8.4	16	36.19	7.3	73.19	13.67
C8.5	2	-	-	-	-
C8.6	2	-	-	-	-
SC1	11	39.09	7.63	67.64	15.86
SC2	7	34.29	3.86	63.57	5.97
SC4	4	-	-	-	-
SC5	2	-	-	-	-
SC7	1	-	-	-	-
SC9	3	-	-	-	-
CCS1	1	-	-	-	-
CCD	72	-	-	73.32	7.74
COD	1	-	-	-	-

The tables that follow give the question statistics for each paper for Mathematics candidates. Data for papers with fewer than six candidates are not included.

Paper C1.1: Model Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.33	19.33	1.63	6	0
Q2	16.67	19.2	6.59	5	1
Q3	15.75	17.43	7.15	7	1

Paper C1.2: Gödel's Incompleteness Theorems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	23.75	23.75	1.83	8	0
Q2	22.5	22.5	2.88	6	0
Q3	25	25	0	2	0

Paper C1.3: Analytic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	22.62	22.62	2.43	13	0
Q2	23.11	23.63	1.96	8	1
Q3	22.93	22.93	2.79	15	0

Paper C1.4: Axiomatic Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.14	16	5.81	6	1
Q2	17.88	17.88	3.04	8	0
Q3	14	14	4.90	4	0

Paper C2.1: Lie Algebras

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.67	17.67	4.76	6	0
Q2	19.88	19.88	2.30	8	0
Q3	19.5	19.5	0.71	2	0

Paper C2.2: Homological Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	22.33	22.33	1.75	6	0
Q2	21	21	4.20	7	0
Q3	16.8	16.8	4.44	5	0

Paper C2.4: Infinite Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.71	15.71	2.06	7	0
Q2	17.5	17.5	4.14	6	0
Q3	22	22		1	0

Paper C2.5: Non-Commutative Rings

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.67	19.67	2.31	3	0
Q2	14.5	16	2.12	1	1
Q3	19.5	19.5	3.79	4	0

Paper C2.6: Introduction to Schemes

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	23.33	23.33	0.58	3	0
Q2	22	22		1	0
Q3	20.5	20.5	0.71	2	0

Paper C2.7: Category Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.2	15.2	2.53	10	0
Q2	21.3	21.3	2.16	10	0
Q3	19	19	2.83	2	0

Paper C3.1: Algebraic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	23.2	23.2	1.92	5	0
Q2	20.25	20.25	5.25	4	0
Q3	23	23	0.71	5	0

Paper C3.2: Geometric Group Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.67	20.67	2.08	3	0
Q2	13.5	13.5	4.95	2	0
Q3	24	24		1	0

Paper C3.3: Differentiable Manifolds

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14	14	9.90	2	0
Q2	17	17	3.81	5	0
Q3	14.67	14.67	3.21	3	0

Paper C3.4: Algebraic Geometry

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.75	21.75	2.92	8	0
Q2	22.25	22.25	3.59	4	0
Q3	19.5	19.5	3.42	4	0

Paper C3.5: Lie Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.6	20.6	2.51	5	0
Q2	20.5	20.5	6.36	2	0
Q3	22	22	3.61	3	0

Paper C3.7: Elliptic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.86	21.86	4.30	7	0
Q2	18.29	18.29	6.85	7	0
Q3	20.42	20.42	2.94	12	0

Paper C3.8: Analytic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.93	19.00	6.79	14	1
Q2	17.53	17.53	5.29	15	0
Q3	8.6	8.6	3.51	5	0

Paper C3.10: Additive and Combinatorial Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.91	19.91	6.59	11	0
Q2	16.27	16.27	9.33	11	0

Paper C4.1: Further Functional Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.67	15.67	4.89	6	0
Q2	15.33	15.33	5.35	6	0
Q3	14.75	14.75	4.99	4	0

Paper C4.3: Functional Analytical Methods for PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.75	16.75	1.26	4	0
Q2	20.5	20.5	0.71	2	0
Q3	17.25	17.25	5.85	4	0

Paper C4.6: Fixed Point Methods for Nonlinear PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.33	17.33	2.52	3	0
Q2	18.4	18.4	3.21	5	0
Q3	17.5	17.5	9.19	2	0

Paper C4.8: Complex Analysis: Conformal Maps and Geometry

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.25	11.25	2.22	4	0
Q2	17.75	17.75	2.06	4	0
Q3	20.5	20.5	2.12	2	0

Paper C5.1: Solid Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	22.5	22.5	2.38	4	0
Q3	17.25	17.25	4.99	4	0

Paper C5.2: Elasticity and Plasticity

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.42	17.42	4.48	12	0
Q2	15.14	15.14	8.07	7	0
Q3	17.18	17.18	6.81	11	0

Paper C5.5: Perturbation Methods

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17	17	3.46	6	0
Q2	19	19.2	4.37	15	1
Q3	19.93	19.93	3.51	15	0

Paper C5.6: Applied Complex Variables

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.8	19.8	3.94	10	0
Q2	19.83	19.83	0.98	6	0
Q3	21.9	21.9	2.51	10	0

Paper C5.7: Topics in Fluid Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.8	16.8	1.92	5	0
Q2	17.125	17.13	5.17	8	0
Q3	19.8	19.8	3.90	5	0

Paper C5.9: Mathematical Mechanical Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.25	17.25	2.63	4	0
Q2	16.33	16.33	3.21	3	0
Q3	23	23		1	0

Paper C5.11: Mathematical Geoscience

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.2	20.2	3.26	10	0
Q2	18.71	18.71	3.31	17	0
Q3	19.86	19.86	5.30	7	0

Paper C5.12: Mathematical Physiology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	21.5	21.5	2.38	4	0
Q2	15.33	15.33	1.15	3	0
Q3	23.33	23.33	1.53	3	0

Paper C6.1: Numerical Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.6	18.6	2.51	5	0
Q2	19.5	19.5	1.73	4	0
Q3	17.67	17.67	3.51	3	0

Paper C6.2: Continuous Optimization

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.67	18.40	2.58	5	1
Q2	17.8	17.8	2.49	5	0
Q3	15.88	15.88	3.44	8	0

Paper C6.3: Approximation of Functions

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	24	24	0.82	4	0
Q2	17	17	-	1	0
Q3	16.67	16.67	8.50	3	0

Paper C6.4: Finite Element Methods for Partial Differential Equations

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.2	16.2	4.38	5	0
Q2	16.89	16.89	3.66	9	0
Q3	16.5	16.5	5.80	4	0

Paper C7.4: Introduction to Quantum Information

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20	20	4.36	3	0
Q2	20.67	20.67	5.77	3	0
Q3	20	20	5.66	2	0

Paper C7.5: General Relativity I

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.5	18.5	9.19	2	0
Q2	16	16	-	1	0
Q3	19	19	-	1	0

Paper C7.6: Relativity II

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15	15	-	1	0
Q2	8	8	-	1	0

Paper C8.1: Stochastic Differential Equations

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.2	19.2	3.52	10	0
Q2	13.67	13.67	3.51	3	0
Q3	20.14	20.14	6.04	7	0

Paper C8.2: Stochastic Analysis and PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16	16	3.52	6	0
Q2	17	17	1.41	2	0
Q3	18.88	18.88	5.03	8	0

Paper C8.3: Combinatorics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.19	16.19	5.71	16	0
Q2	11.5	13	3.11	3	1
Q3	19.35	19.35	4.86	17	0

Paper C8.4: Probabilistic Combinatorics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.4	19.4	2.75	15	0
Q2	17.29	17.29	4.82	7	0
Q3	16.7	16.7	5.08	10	0

Paper C8.5: Introduction to Schramm-Loewner Evolution

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15	15		1	0
Q2	14	14		1	0
Q3	19.5	19.5	4.95	2	0

Paper C8.6: Limit Theorems and Large Deviations in Probability

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	25	25	0	2	0
Q2	20.5	20.5	3.54	2	0

Paper SC1: Stochastic Models in Mathematical Genetics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.63	17.43	6.21	7	1
Q2	21.73	21.73	2.80	11	0
Q3	17.25	17.25	4.35	4	0

Paper SC2: Probability and Statistics for Network Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18	18	3.21	7	0
Q2	15.4	15.4	2.30	5	0
Q3	18.5	18.5	2.12	2	0

Paper SC4: Statistical Data Mining and Machine Learning

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.5	20.5	5.80	4	0
Q2	16.67	16.67	6.11	3	0
Q3	19	19	-	1	0

Paper SC5: Advanced Simulation Methods

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	25	25	0	2	0
Q2	23.5	23.5	0.71	2	0

Paper SC7: Bayes Methods

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q2	24	24	-	1	0
Q3	23	23	-	1	0

Paper SC9: Interacting Particle Systems

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.33	20.33	4.04	3	0
Q2	21.67	21.67	1.15	3	0

D. Recommendations for Next Year's Examiners and Teaching Committee

Examiners feel that the department should reconsider the rule on re-scaling of the 50 raw mark , and how this information should be communicated to assessors- i.e. 50 raw mark is for outstanding candidates and that the work for these candidates should significantly show this.

E. Comments on papers and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. Some data to be found in Section C above have been omitted.

C1.1: Model Theory

Q1. Parts a-c were generally well done. Part d required reproving the Los-Vaught criterion in this setting, rather than applying it. Very few saw this. A very common mistake was to assert that T' must be categorical, with no explanation; there is no reason it should be.

Q2. (a) was done well almost universally, including the explanation of where completeness is used. (a) was done well almost universally, including the explanation of where completeness is used. (b,c) were also generally handled well; occasionally the wrong answer was given regarding aleph-one categoricity (it need not hold.) In 2d, there are at least two routes to a correct solution, one invoking homogeneity of the countable models of an aleph-zero categorical theory, the other using Ryll-Nardjewski. The latter was chosen by most, and on the whole people did well. Some attempted a direct approach, quoting neither theorem, and a few succeeded; among the others, a surprisingly common mistake was to claim that permuting two elements while keeping the rest fixed is an isomorphism of an arbitrary structure.

Q3 This was the most popular problem. On the whole people dealt very well with the new notions of irreducibility and JEP, and proved them equivalent (3bc). 3d was seen as more difficult; only about half of those choosing Q3 attempted it; in general they did well. JEP or irreducibility show that two existential sentences, each of which hold in some model, can hold simultaneously in some model; this easily extends to finitely many existential sentences; then either a limit or a compactness argument is required.

C1.2: Gödel's Incompleteness Theorems

This year the section C exams, like everything else, were affected by the Covid-19 pandemic. The colleges and university buildings were closed, and students sat their exams remotely, online. They had extra time, because of downloading papers and then uploading their completed scripts, they were allowed to use books and notes, and they took fewer papers each. Thus "bookwork" parts of questions became tests of understanding rather than memory. But the answers that candidates gave leave the impression of a student body that was committed, attentive, able, and hard-working, and the answers were on the whole very good.

Question 1. was very well done on the whole, though, judging by the overall appearance of scripts, some candidates may have spent too much time on it and run out of time on their other question.

Parts (a)(i) and (a)(ii) of question 1. were done very well. In part (a)(iii), one is using the Gödel sentence, that asserts its own unprovability. Part (a)(iv) is asking, in effect, for a proof that any Π_1 sentence that is neither provable nor disprovable from the first-order Peano axioms must be true in the natural numbers. Some candidates misunderstood this.

The sentences in part (b) are all variations on the Gödel sentence, and the sub-parts of part (b) may be tackled in a number of ways: either by making the relationship between the various sentences H_i and the Gödel sentence explicit, or by treating them individually.

The permitted assumptions given at the beginning of the paper in fact imply that the first-order Peano axioms are 1-consistent (a fact which is needed from time to time). Candidates varied according to how carefully they verified this.

Question 2., which concerns provability logic, turned out to permit many differences of approach.

Part (a) was done well on the whole, but there was some vagueness about the rules of inference in GL-logic. There is an important difference between the rules Modus Ponens and Generalisation: Modus Ponens says that if ϕ and $(\phi \to \psi)$ occur earlier in a proof then it is legitimate to put ψ later on (and none of these three formulae need be theorems of GL-logic, if the proof involves some extra assumptions), whereas Generalisation says that if ϕ is a theorem (i.e. is provable without extra assumptions), then $\Box\phi$ is a theorem also. This difference in how the rules may be applied, wasn't always appreciated.

Passing over part (b)(i) (which was done well), there were many attempts at (b)(ii) which varied in how successful they were, but quite a few different functions v were made to work perfectly. The idea is that v should be some kind of (explicitly-defined) valuation, where the notion of "valuation" used is sufficient that all theorems of GL-logic end up given the value of "true". Part (b)(iii) is a straightforward application of part (b)(i).

There were many different successful strategies for finding fixed points in part (c). Some people followed the algorithm in the lecture notes, thus implicitly using the fact, which follows from theorems in the notes, that this algorithm is correct. Others found answers by some method or other (intelligent guesswork, or other methods done in rough and then consigned to the bin?) but proved they were correct by arguing, for each fixed point F, that $A(F)$ and F were provably equivalent.

Question 3. was also done well, though the quantity of deleted text in the submitted solutions suggests that some candidates found it quite hard.

Part (a) was relatively easy going. In part (iii), quite a few candidates were unwilling to simply assume that any Π_1 sentence was Σ_2, and some worked harder than was necessary to prove this. A trivial proof goes like this: if ϕ is explicitly Π_1, and x is a variable which does not occur free in ϕ, then ϕ and $\exists x\, \phi$ are equivalent; and $\exists x\, \phi$ is explicitly Σ_2.

Parts (b) and (c) are about the weird behaviour of ω-consistency.

Part (b) caused some difficulty. The point of part (i) is that any set of sentences true in the natural numbers must be ω-consistent (think about the rules concerning the semantics of existential quantifiers), so X can't possibly be true in the natural numbers; but then X, when added to the first-order Peano axioms, gives an ω-consistent system (which can only be true in non-standard models of the Peano axioms, containing many non-standard elements which could witness the truth of existential statements). Part (ii) was done well.

Part (c) caused difficulty. Looking at part (i), the Second Incompleteness Theorem is in the notes; and the non-provability of the consistency of the Peano axioms follows because consistency is the unprovability of contradictions; and it follows from the Second Incompleteness Theorem that nothing can be proved to be unprovable. As for part (ii), consistency follows from part (i), Σ_1-unsoundness follows because $\neg\, \mathrm{Con}_{\mathrm{PA}}$ is Σ_1 and untrue, and then ω-inconsistency follows from that.

C1.3: Analytic Topology

This year, everything at the University was affected by the Covid-19 pandemic. The university buildings and colleges were shut for Trinity term, and exams were sat remotely, online, and were open-book, meaning that candidates could use books and notes during the exam. They were also given more time, to allow for downloading the exam and uploading the finished scripts.

This, in my view, made unavoidable the perennial question of what exams are for and what they should be designed to measure. The open-book format, I think, changed the emphasis from knowing material (having committed proofs to memory), to understanding it: failures in understanding can reveal themselves in significant errors in reproducing arguments.

There were some very high raw marks this year. Various explanations for this come to mind; but one is that very many of the students were able, attentive, and diligent, and that this was reflected in their scripts.

In question 1.(a), which was bookwork, there were many very good answers. Where people fell down was in forgetting to do part of the argument, for example, neglecting to prove that some particular kind of open set had open preimage under the function $f_{C,D}$. In bookwork generally, this is an easy mistake to make. But it's also quite serious, and easy to guard against: write a list of the things you need to do, and tick them off one by one as you do them.

In 1.(b), to which many good answers were given, a common problem was, in part (ii), defining an operator $H_X(x, U)$ for the subspace X which was actually monotonic in U. People who paid careful attention to the hint did not make this mistake, but people who reasoned as follows did: "Let U be an open subset of X, and let x be an element of U. *Then there exists and open subset* U' *of* Y *such that* $U = X \cap U'$. Define $H_X(x, U)$ to be

$X \cap H_Y(x, U')$." The reason this doesn't work, of course, is that if $U \subseteq V$, we have no guarantee that $U' \subseteq V'$. In part (iii), some people neglected to prove that $\overline{H(C, D)} \subseteq X \setminus D$, which is of course rather important.

There were quite a few good answers to 1.(c).

In 2.(a), which is bookwork, there was again a problem with people forgetting to describe vital steps in the argument.

The commonest difficulting in 2.(b) was in proving that Z had a countable basis of clopen sets and that, moreover, Z had just countably many clopen sets. Some people tried to do this and failed; some people didn't do it at all, failing to note that even though Z has a countable basis, and also has a basis of clopen sets, it still requires to be proved that there are just countably many clopen sets in Z.

Part 2.(c) was done well on the whole.

There were a lot of good answers to 2.(d), but some people either couldn't do it or doubted that the result was true.

I think people generally spotted that in parts (b) and (d) of question 2., they needed to adapt the proof in part (a).

Part 3.(a) was generally done well, but just as in part (a) of the other two questions, it was possible to forget to do a part of the proof. For example, one or two people forgot to prove that $h(X)$ was dense in αX.

Many people did 3.(b) well. Some wasted a minute or two in proving that αf was proper, which the question did not ask them to do.

Parts (i) and (ii) of 3.(c) were generally done well. People gave different answers to (ii), all of which I deemed to be correct. For example, some people gave a definition that worked with ultrafilters and onto maps, and others gave more general definitions in terms of general filters and maps that might not be onto.

There were quite a few good attempts to do part (iii) of 3.(c). Some people had difficulty with it; I can't detect a general explanation as to why that's more sophisticated than saying that the argument was a bit tricky and a bit fiddly, and it was possible to get lost in the set algebra.

C1.4: Axiomatic Set Theory

Question 1 As expected, candidates were able to do the bookwork and the common applications. Often counterexamples were missing and a significant number of candidates claimed that functions from a to b are elements of $a \times b$ instead of subsets. This led to wrong answers for parts (b)(iii) and (iv). Almost no candidate noticed that (b)(iii) for transitive models of **ZFC** is extremely similar to the absoluteness of the set of finite functions with given co-domain.

Question 2 Marks in this question were high because the proof of the Reflection Principle was done very well. In (b)(ii) candidates usually demanded only one of the absoluteness of ϕ and $\exists y\ \phi$ instead of both (or rather their conjunction) and thus gave partially incorrect answers. In (c)(ii) almost no candidate spotted that V_α is definable in V with parameter α despite having written exactly such a definition in (a).

Question 3 The somewhat technical reasoning in (a)(ii) and (iii) proved more challenging than expected. No candidate explained why $Def(x)$ is a set (in **ZF-Powerset**) and describing the extension of a well-order of x to $Def(x)$ often contained mistakes or was missing.

In (b) a large number of candidates showed transitivity of M by going through cases instead of observing that $y \in x \in M_n$ gives $y = \bigcup\{y\} \in M_{n+2}$. Explaining why M was a model of **ZF-Replacement-Powerset** was done well, though some candidates missed the occasional axiom.

Very few candidates followed the hint and showed that $TC(x) \cap \omega \times \omega$ is finite. The others made little progress.

C2.1: Lie Algebras

Question 1: This was a popular question but part (d) was challenging and only two candidates submitted full solution. Part (i) was the main stumbling block: Use that $\mathfrak{g} = [\mathfrak{g}, \mathfrak{g}]$ together with the fact that $f(ad([x,y])) = [f(ad(x)), ad(y)] + [ad(x), f(ad(y))]$ is an inner derivation by (b).

Question 2: This was the most popular question with many good almost complete solutions. In (c) some candidates gave an alternative proof using that $ad(y)^2(x) = 0$ for every eigenvector y of $ad(x)$.

Question 3: Few attempts with two nearly complete solutions. In part (d) some candidates did lots of calculations justifying why the root system is no larger than the 48 roots they have already found and missed the intended argument: Since each orbit of W on Φ intersects B it follows that the length of each root is either 1 and 2. Together with $\Phi \subset \mathbb{Z}B \subset \frac{1}{2}\mathbb{Z}^4$ this easily gives that Φ cannot have more than 48 roots.

C2.2: Homological Algebra

Question 1 was done better than Question 2 and 3, but I really don't think that it was much easier. The fact that it was done better might be a result of the students attempting it first, and thus feeling less the pressure of time. I was particularly pleased by how well the students answered the last part (part (d)) of Question 1. Few students, however, managed to do d(iii).

C2.3: Representation Theory of Semisimple Lie Algebras

Question 3 was done well, and was perhaps straightforward, given basic understanding about the Weyl Dimension Formula by the candidates. Question 2 was more tricky. Pitfalls included the claim that the tensor product (with the diagonal action) of two simple modules is simple; this is false as the tensor square of any module of dimension ¿ 1 admits a non-trivial decomposition into the even part and the odd part. Question 1 was less popular.

C2.4: Infinite Groups

Question 1: All candidates attempted this question, and have displayed solid knowledge of the basics on nilpotent groups. In the third part of the question, a surprising proportion of the candidates failed to find the exact nilpotency class of the group, even after showing that the lower central series of the subgroup H ended one step earlier than that of the group. As surprisingly, there was no complete answer to the fourth part of the question, even though the hint almost gave away the answer.

Question 2: Almost all candidates chose this second question as well and have been altogether more successful than in Question 1. The arguments in the second part of the question were at time lacking rigour, or were missing steps. This may show a tendency to pay more attention to the abstract theory and less to the worked examples.

Question 3: Hardly anyone attempted this question. This question covered the last part of the course, which may explain its lack of popularity.

C2.5: Non-Commutative Rings

Question 1: This was a popular question whith several nearly complete answers. The common mistakes were in (c)(i) when analysing a general element from $k[G](H-1)$

Question 2: Not very popular with just 2 attempts. Parts (a) and (b) were done well, but there was no successful attempt on (c).

Question 3: This question was attempted by all candidates with good results. Common errors were failing to to explain why the graded ring of the Weyl algebra is commutative in (c) or failing to define a good filtration on the module M in (d). There were good solutions to (e) with only minor gaps.

C2.6 Introduction to Schemes

All candidates picked Q1, and then almost evenly split in choosing Q2 or Q3. The scripts were all of an exceptionally high standard, with no raw marks below 20 in any question. In Q1(c) candidates correctly stated what the functor does on objects, but often forgot to state what the functor does on morphisms. In Q2(c) candidates did not use the Hint that implies that X is the union of those basic open sets. In Q3(b), when checking the sheaf property, several candidates did not exploit the Hint: for integral schemes the restriction maps on functions are injective, and overlaps are non-empty by irreducibility. In Q3(c) candidates correctly stated which module works locally, but forgot to justify why it works.

C2.7 Category Theory

There were plenty of answers demonstrating very good understanding of the material, though none was perfect.

Question 2 was the most straightforward question and there were attempts from almost all the candidates, some of which were very good though none obtained full marks. There was a minor misprint in 2 (b) (iv), where a superscript 'op' was missing in $\mathrm{Hom}_{\mathrm{Cat}}(\mathcal{C}^{op}, \mathrm{Set})$, but it seemed clear that it did not affect any candidates adversely; indeed only one gave any indication of noticing it.

Question 1 contained a more substantial amount of unseen material. There were many very serious attempts, and each part of the question was answered correctly by several candidates, but no candidate gave a completely successful solution to the whole question. In 1(a) it was disappointing to see that, just as last year, few candidates could give an accurate construction for coequalisers in Set.

Question 3 did not attract many attempts. The bookwork part of the question was answered well, but the rest of the question, especially (b)(iv), caused difficulties.

C3.1: Algebraic Topology

Almost all candidates picked Q1, and then about two thirds of students picked Q3 instead of Q2. The standard of almost all scripts was very high. In Q1(b) some candidates did not carefully show functoriality. In Q1(c) there is a difference in the homology and cohomology calculations, but some leniency was allowed if candidates checked the homology version of the Kunneth theorem instead of the cohomology version. Q2(c) candidates sometimes failed to carefully explain why the cup product lands in the correct chain complex. Q2(d) requires a commutative diagram, relating the map from (c) with the usual cup product on X, and only one candidate did this correctly. In Q3(d) some candidates did not carefully justify why the preimage of a point is a finite set. Almost no candidate explained in a satisfactory way why the local degrees are all +1. They did not notice that the previous step in (c) had built the pull-back orientation precisely so that the orientation generator of X maps to that of Y, locally.

C3.2 Geometric Group Theory

Question 1: This was attempted by all candidates, with fairly complete answers provided, in particular for the examples in the second part. Only half of the candidates attempted the question where the isomorphisms between two groups was to be proven by finding a finite sequence of Tietze transformation, and that seems to show that while the formal knowledge of this method was acquired, the intuition behind it is still lacking.

Question 2: This question was for the main part purely theoretical. Nevertheless, while most students displayed confident knowledge of everything related to the fundamental group of a graph of groups, very few attempted the second part of the question, requesting to provide a definition of such a group for finite simplicial trees *via* a universal property.

The third part on residual finiteness was answered reasonably well.

Question 3: This question was attempted in equal measure as Question 2, and answered very well on the whole, including the parts requiring an algorithmical approach. Surprisingly the least well answered part was part (b), in which a simple geometric argument, relying entirely on the geometry of hyperbolic spaces, was all that was needed.

C3.3: Differentiable Manifolds

Question 1. Most students lost a mark on part (a) because they missed out some detail in the definitions, typically in the definition of the free and properly discontinuous action of a group by diffeomorphisms. Part (b) was mainly done well, with a few students losing marks through small slips in showing that the volume form defines an orientation. Almost all students had the right idea for (c)(i), but most commonly lost marks for not showing that the orbits of the open sets they chose were disjoint, and for not observing that $f_r(z) \in \mathcal{S}^3$ for $z \in \mathcal{S}^3$. Part (c)(ii) was similar in that most students had the right idea and approach, but lost marks through lack of details; a common error was not showing that f_r is orientation-preserving correctly, and not showing that the 3-form defined on the quotient is nowhere vanishing. About half of the students who attempted this question had the right idea for (d), all using $M \cong \mathbb{RP}^2 \times \mathcal{S}^1$ as the example, but did not fully justify their answer. This was the least popular question and produced a wide spread of marks, from high to low.

Question 2. Part (a) was done well. A number of students dropped marks in (b), typically by not observing that the putative vector fields and/or diffeomorphism they constructed are smooth. Part (c) was mainly done very well, but students lost marks in not explaining that for a diffeomorphism the induced map on de Rham cohomology is an isomorphism.

Part (d)(i) was done very well by almost all students, with marks only lost for not referring back to the criterion for parallelizability in (b) and for not checking the homotopy they construct is well-defined. Whilst most correctly students correctly understood that $\mathcal{S}^2 \times \mathbb{R}$ is parallelizable in (d)(ii), most did not justify it adequately. This was the most popular question, with all but one student attempting it. There was a spread of marks, but no low marks (below 10).

Question 3. Part (a) was usually done well, though some students dropped marks by not stating the criterion for a map to be a local diffeomorphism clearly. Most students dropped a mark on part (b), usually by not defining the interior product. Part (c)(i) was done well, except some students thought the flow was a vector field, even though they defined it correctly in (a). Part (c)(ii) created a great variation in responses, with most difficulties arising in the correct computation of the Lie derivative from the definition, but also in computing the exterior derivative and the interior product correctly. In part (d), most students had the right idea, but some made errors in their computations. There was a spread of marks, but also no low marks (below 10).

C3.4: Algebraic Geometry

The paper was done very well by 12 candidates.

Question 1 had 11 answers at an average of 21 marks. Candidates lost some marks on imprecise explanations of easy checks. In part (d) some candidates explained what are the closed subvarieties of $\mathbb{A}^1$ but missed identifying the irreducible ones. In part (e) some candidates thought $(0, 0)$ was a separate component; others failed to prove that their proposed components are indeed irreducible. One of the equations in (e) had an obvious misprint that was corrected by all candidates (or not even noticed).

Question 2 had 8 answers, also at an average of 21 marks. Most of those who lost marks did so on the computation in (e). There is in fact a condition missing in (e), in that it should be assumed that the quadrics define a reduced ideal; most candidates tacitly assumed this.

Question 3 had several answers lacking a full explanation in (b) of the fact that the map π was a morphism of quasiprojective varieties. Some candidates failed to check that in (c)(i) their proposed inverse does indeed provide an inverse.

C3.5: Lie Groups

Question 1

This question was about the noncompact symplectic group $Sp(2n, R)$. It was the most elementary question in terms of the material covered, and all 8 candidates attempted it, with the majority getting marks in the 18-25 range. Most candidates got through the proof that the symplectic group was in fact a Lie group, though some were careless about quoting the appropriate theorems to justify this. The parts of the question aimed at understanding the algebraic structure of the group were generally done well, though only a couple of candidates really gave a good explanation of the geometric interpretation of the isomorphism in the $n = 1$ case.

Question 2

This question, on representation theory, proved less popular. The standard parts of the questions were done well, but putting everything together to show nonexistence of nontrivial finite-dimensional unitary representations of $SL(2, R)$ proved more challenging.

Question 3

This question was on Weyl groups and maximal tori with candidates attaining marks in the 18-25 range. Candidates generally had a good grasp of the maximal torus for $SO(2n + 1)$ and how to find Weyl group elements. The more subtle case of $SO(2n)$ was of course harder, but some people understood this well.

C3.7: Elliptic Curves

Question 1: This was attempted by 9/17 candidates and done very well, with only a few students losing marks on parts (b), (c) and (d).

Question 2: This was taken by 10/17 students. Few had problems with (a) and (b). Part (c) was rather hit and miss, with half the students spotting the equation did not actually define an elliptic curve (such curves cannot have infinitely many integral solutions) and scoring highly, and the remainder making no progress. I was impressed that the majority of students were able to work out part (d).

Question 3: A predictably popular question, taken by 15/17 candidates. High marks overall, the main problem being occasional computational mistakes at crucial moments led some students astray.

C3.8 Analytic Number Theory

Question 1: This question was very popular, being attempted by almost all candidates and answers were generally good, typically scoring higher than the other questions. All answers unsurprisingly gave correct answers to (a) which was bookwork. For part (b) most answers had little difficulty in applying Poisson summation to prove the functional equation for the theta function, and either candidates spotted how to answer the later parts or missed them. There were some slips in relating the theta function to the zeta function, even though the final part of the question could have been answered by just following the notes. The high average quality of the answers and the popularity indicate that (at least for the open book format) this question was close to being too straightforward, but fortunately it produced a reasonable spread of marks to distinguish between candidates.

Question 2: This question was also popular, and generally answered reasonably well. Beyond some minor slips, the straightforward parts (a), (b) and c(i) were answered well. Several candidates forgot to verify that the polynomial in c(ii) had a non-zero leading coefficient. It was pleasing to see that virtually all candidates were comfortable with the strategy of part (c), even if there were some difficulties of execution. A common issue was trying bound the I_2 integrals by taking the largest value of x^s and $\zeta(s)$ separately, which gave too large a contribution. Candidates struggled much more with (d), which typically differentiated between the stronger scripts and the weaker ones. The level of this question felt correct, and worked well even in the different format.

Question 3: This question was not well answered and not popular with the candidates, being attempted by less than one third of all scripts. The candidates which did attempt this tended to do reasonably at the earlier parts, but failed to realise that the later parts were applications of partial summation or the Prime Number Theorem, meaning that they struggled to make much headway. The original version of this question had a much larger bookwork component, but this was replaced by computations involving partial summation with the change to open book format. In retrospect at least, this made the question too hard. Perhaps the ideas should also be addressed more in problem sheets in the course, since I was surprised quite how much candidates struggled. The question distinguished between candidates which answered it, but was clearly a bit too difficult in comparison with the earlier questions.

Summary: Overall this exam adequately distinguished between candidates, despite the challenges of the open book format. Questions 1 and 2 worked well, but question 3 was rather less successful. Overall the exam wasn't as well balanced as last years exam, although it is difficult to tell how much of this is due to the change in exam format. Candidates generally showed good understanding of the key ideas of the course, with the exception being a weakness in applications of partial summation which should be noted going forwards.

C3.10: Additive and Combinatorial Number Theory

Fourteen candidates took the exam, and all of them elected to answer questions 1 and 2. This meant that no candidate attempted a question on the additive combinatorics part of the course.

Questions 1 and 2 were done quite well, with around a third of the students submitting almost perfect solutions and several other candidates solving significant parts of questions.

In Q1, the most common mistake was to forget the need to assume a coprimality condition in the bound for Gauss sums, and therefore forget, in 1 (b), to divide up into powers of p (as in the notes).

In Q2, the most common mistake was to forget to handle the case $h = 0$ separately in part (b) (or to do something equivalent).

There were a few different solutions to Q2 (c), all different to the official solution, which was nice to see.

C4.1: Further Functional Analysis

As this was an open book exam, bookwork material was reduced to a minimum. Core material in the form of standard arguments in new settings was well answered by the vast majority of candidates. Each question included genuinely stretching material which most candidates struggled with, but there were some exceptional scripts showing complete command of the material in the course, tackling this challenging material with aplomb.

Question 1

This was marginally the most popular question. Part (a)(i) and (ii) was very well answered, but while a number of candidates identified suitable spaces in which an example for (a)(iii) might be found, only one candidate was able to turn this intuition into a precise argument. The Zorn's lemma argument of (b)(i) was well handled, but while candidates started the separation argument required for (b)(ii) well, they often didn't identify a suitable element to adjoin to M to contradict maximality.

Question 2

Part (a) was broadly well answered, though rarely in a completely optimal fashion leading to lost marks for imprecisions. Candidates tackled (b) well, with many candidates giving good arguments for (b)(i) and all candidates correctly using Schauder's theorem in part (b)(ii). Part (c) was found to be challenging with only one candidate finding a complete argument. Candidates often failed to find an appropriate set to apply the Arzela-Ascoli theorem to (namely the compact set $\{f_n : n \in \mathbb{N}\} \cup \{0\}$) and often got sidetracked.

Question 3

Part (a) was generally very well answered, with most candidates finding the inverse mapping theorem argument needed in (a)(ii). In (b), part (i) and (ii) where well done, but (iii) proved harder with candidates finding it difficult to set this up precisely, often attempting to define a projection directly on X rather than on $Y + Z$ as indicated in the question. Part (c) is related to the bookwork result that $T + S$ is Fredholm when T is Fredholm and S is compact; the key part of this argument is to show that $T + S$ has closed range. In the set up of 3(c), while the operator T need not be Fredholm, only a few candidates realised that the argument that $T + S$ has closed range goes through in the same fashion.

C4.3: Functional Analytic Methods for PDEs

Question 1: This problem was popular and was handled with variable degree of success. In (b), some candidates made an error when making the change of variables $y = \frac{x-x_m}{r_m}$. In (c)(ii), some candidates showed only the weaker statement that for a given compact subset ω of $\bar{D} \setminus \{0\}$, there is a subsequence h_{m_k} which converges strongly in $L^q(\omega)$.

Question 2: Only about two fifth of the candidates attempted this question. The question was well answered except for (c)(ii). The candidates forgot that the vector field $(-x_2, x_1)$, though is smooth, does not belong to $L^p(\mathbb{R}^2; \mathbb{R}^2)$.

Question 3: This problem was most popular. Most candidates had some difficulty in solving (a)(ii), though understood that it was related to the Fredholm alternative. About half of the candidates had some difficulty in solving (b)(ii), where Rellich-Kondrachov's theorem was expected to be used.

C4.4 Hyperbolic Equations

Question 1: Part a) of the question was answered correctly by nearly everyone, however the bookwork parts b) and c) posed considerable difficulties for the candidates. The construction of the unique entropy solution in part d) was attempted by all the candidates, but everyone made a mistake at different stages and none succeeded in finding the correct equation for the first shock curve. Some candidates got very caught up in wrong computations and lost a lot of time here which was clearly lacking for the later questions.

Question 2: All candidates struggled with the proof of the one dimensional Sobolev embedding in part a)(i) of the question and the energy estimate in part a)(ii). Part b) was received well; the candidates had in general a correct approach to the solution however some did not make use of the a priori boundedness of the solution to conclude the argument.

Question 3: Part a)(i) of the question did not pose any difficulties, the other parts were not attempted probably because of lack of time.

C4.6 Fixed Point Methods for Nonlinear PDEs

Question 1: Question 1 was solved by slightly less than half the candidates. Parts a) and b) were solved very well and high marks were achieved on those parts. As expected Part c) was more difficult. 1c)i) was in general well done, though solutions were quite long as none of the candidates realised that the amount of discussion needed could be significantly reduced if one states the first version of Schauder rather than any of the other versions. For the second part of c) most candidates realised that a proposition from the lecture was useful as one can use the condition to show that the sign of the inner product is constant outside a suitable ball, but no one could provide a counterexample for the very last bit (e.g. f defined by $f(x) = x$ in the unit ball and $f(x) = \frac{x}{|x|}$ outside the unit ball works). 1c)iii) was designed to be the most difficult part of the question and while no complete solution was provided one of the students came close by suggesting the use of a shift to prevent the existence of a fixed point (though some extra modification is then needed to make sure that S maps into H^1.)

Question 2: Question 2 was solved by all candidates. High marks were obtained on 1a), though quite a few students did not realise that since the existence of a unique weak solution in (ii) could be assumed, all that needed to be done was to use the solution itself as a test function to derive an estimate, rather than replicating the full proof of existence, uniqueness and continuous dependency of solutions.

The main part (i) of Question 2b) was similar to many exercises solved and caused a surprising amount of problems. Some students did not make any use of the sub and supersolution to define the set M, which then makes it impossible to use the weak maximum principle. The simplest way to approach the problem is to work on the subset of functions in $L^2(\Omega)$ with $\underline{u} \leq u \leq \bar{u}$ and while some students took this route, quite a few worked in H_0^1 instead where one needs to further restrict the set of function to obtain boundedness of M. Nearly all students realised that the second part is an immediate consequence of the result of (i) and Schauder's Fixed Point theorem and got the corresponding 1 mark. The third part was then designed to test whether the students could reduce a new problem to a just proven result and as expected the result was mixed with most students either getting all three points or no marks. Part c) was then very well solved by most students.

Question 3:

Question 3 was solved by slightly more than half of the students and the results were quite mixed but consistent with the performance on question 2. The main comment to make on question 3 is that students did not make good use of the results that they were told they could use without proof and spent a lot of time proving results that were unnecessary.

Part a) was generally well solved, though most students did not comment that in order for their definition of hemi-continuity to make sense one need to assume that the subset is convex. The first part of b) was testing the understanding of one of the basic concepts of the lecture and was generally well solved. Many students wasted a lot of time trying to prove part (ii) from scratch rather than arguing that since the domain is bounded X embeds continuously into $Y = W_0^{1,p}$ and then using the given result that $A_p : Y \to Y^*$ is a monotone operator. Similarly, several students did not realise that the previous parts already give that the operator one is lead to consider in (iii) is a monotone operator and reproved this. All students attempting 3)c) successfully used Sobolev/Gagliardo-Nirenberg embedding theorem to prove that the operator is well defined. While the very last part of 3c) was designed to be challenging, several students got at least partial marks. The full solution to this question uses non-negative functions to disprove the monotone behaviour of $-B$ and the different scaling behaviour of the two terms to disprove the monontone behaviour of B.

C4.8 Complex Analysis: Conformal Maps and Geometry

Question 1: Only one candidate has not attempted this question. Part (a) is a standard book work that was well done by all students. Part (b) is similar to examples from lectures and problem sheets and was well done by most of the candidates. Part (c) turned out to be challenging, some candidates had the right idea about rescaling but could not complete it. Many candidates started working in a wrong direction, they used t to rotate the function instead of rescaling it.

Question 2: Only one candidate has not attempted this question. Part (a) is mostly a standard book work that was well done by all students. In part (b)(i) some candidates wrongly assumed that the power series for g is valid inside of the unit disc and obtained the result by applying the residue theorem. Only half of the candidates could see how the length estimate from (b)(ii) could be used to prove the isoperimetric inequality.

Question 3: This is by far the least popular question. Part (a) is a standard book work that was not challenging. The only challenging part was in (b)(i). In particular, candidates failed to appreciate that the normal vector also changes under the change of variables.

C5.1: Solid Mechanics

Q1: All students tried this question and did it quite well. The first 15 marks were mostly straightforward and students showed a good understanding on the basics of nonlinear elasticity. Only 1/5 students manage to fully complete the last part that required a better understanding of the material. Q2: The candidates who tried this question did poorly and essentially missed the main point of the question. Q3: Similarly to the first question, almost all students (4/5) tried this question with a decent average but very few students fully understood the problem and managed to solve the last PDE for the unknown function.

C5.2: Elasticity and Plasticity

Question 1 This was a popular question, on which well prepared candidates were able to achieve good marks. The bookwork in part (a) was mostly handled well. The subtle scaling argument in part (b) was found to be tricky, as intended. In part (c), many candidates ignored the effect of the inhomogeneous boundary condition on the solvability condition. Arithmetical accurary was also a frequent problem, often exacerbated by a reluctance to use the hint given. In part (d), there were some very nice sketches and qualitative descriptions, but also some who didn't attempt to analyse the given amplitude equation and simply reproduced the standard pitchfork bifurcation diagram from the lectures.

Question 2 This was the least popular question and had the lowest average mark. Part (a) was all bookwork, but nevertheless caused difficulties for weaker students. In part (b), the standard inversion of the Joukowski transformation was handled well, but finding the image system to enforce the zero-stress boundary condition caused many problems. For those who got that far, part (c) was relatively straightforward.

Question 3 This was a popular question, for which there was a mix of very strong and very weak solutions. The bookwork in parts (a) and (b) was generally done well. In part (c), some candidates made their lives harder than necessary by trying to solve everything in terms of displacements rather than using the compatibility condition from part (b). Most of the candidates who made any headway at all with part (d) managed to calculate the slightly awkward integral. However, failure of some candidates to systematically solve for the stress in the elastic region caused confusion over the correct boundary conditions to impose on τ_{rr}.

C5.5: Perturbation Methods

Q1

While this question was not popular, it was well done in general by those making a serious attempt. Weaker scripts did not show that the second integral for $J(x)$ was $\text{ord}(1/[x\epsilon])$. In addition, many candidates were unable to demonstrate that the higher order terms from the power series expansion of the exponential were consistent with the stated error bound.

Q2

This was a very popular question. In the first part, weaker scripts often did not give correct reasoning for the error estimate though essentially all candidates noted the importance of this result in later parts of the question. Accurately considering the expansions with respect to the intermediate variable in the later parts was frequently the most troublesome aspect of the question for candidates.

Q3

Again, a popular question. The first part on multiple scales was very well done in general. The second part, involving WKBJ expansions, was tackled well in the earlier stages though mistakes generally emerged as the calculation proceeded.

C5.6: Applied Complex Variables

Q1: Part (a) was generally well done, although many candidates failed to explain why the majority of the real axis maps to straight-line segments (only explaining what happens at the pre-images of the vertices). In (b), there were quite a few difficulties with evaluating the constants P, Q and λ. Part (c) was well done. For part (d), a number of candidates did not adequately explain their sequence of conformal mappings, which in some cases was wrong, but many got the correct mapping of the hodograph plane to the upper half plane. The last part was naturally done by those who had successfully found λ in part (b).

Q2: Parts of this question were done well, though some struggled with part (b), and the very last part of (d) was not solved by anyone. Part (a) was fine. In part (b), quite a number of candidates assumed that $\tilde{w}_- = -\tilde{w}_+$ when calculating the integral (as for most examples seen during the course), despite having quoted the correct expressions for those two quantities. Part (c) was done easily by most candidates. In part (d), most realised they could use the result from part (b), and obtain the required result by taking $H = 0$. A few candidates had the right idea for the last part, but chose expressions for $H(z)$ that did not tend to zero at infinity as required.

Q3: This was marginally the most popular question and was done well, although the final answer was found by only a couple of candidates. There were few difficulties with parts (a), (b) and (c), although some failed to adequately explain why the given expression holds on the strip in part (c). For part (d), the Wiener-Hopf argument was explained well by many candidates, but evaluating the integral resulted in many errors keeping track of the square roots and 'i's. A few ended up with the correct expression but multiplied by a complex number (whereas the solution should clearly be real).

C5.7: Topics in Fluid Mechanics

Attempts at the three questions were evenly spread among the ten candidates. Question 1 on groundwater flow was competently done. Only a small number of candidates had any idea about the novel last part. Question 2 on plumes was largely bookwork, but nevertheless tricky, and was a good separator. Question 3 on two phase flow was straightforward but also acted as a good discriminator.

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C5.9: Mechanical Mathematical Biology

Q1. This question was attempted by all candidates. Part (a) was done very well. Obtaining the force and moment balance in (b)(i) caused issues for some, in particular observing that λ_x and λ_y are forces and that the force balance is expressed by $\lambda_x'(s) = \lambda_y'(s) = 0$, though the rest of the question could be completed regardless. Part (b)(ii) was largely done well. The arguments in part (c) followed the same pattern, but required properly adapting the force and moment balance for an extensible beam, which caused an issue for many candidates.

Q2. This question was attempted by most candidates. Part (a) was largely well done.

In part (b)(ii), a few candidates failed to notice the statement that the membrane only resists stretching, and made unnecessary and complicated complication involving bending energy. Part (b)(iii) required careful calculation with asymptotic expansion and energy minimisation; this was done well by a few candidates, though no one was able to successfully compute the correction to the pressure, which required expanding the volume constraint.

Q3. This question was attempted by the smallest number of candidates, but was very well done. The question generally required strong conceptual grasp of the map between different arclength configurations. Part (b) required manipulating the expression for the total length in terms of the growth stretch. Full marks required some small comment on the stem being constantly in compression and thus always losing material due to the growth law. The calculation in part (c) required first solving for the angle θ_1 and then expanding the geometric relation $\frac{dy}{dS_0} = \gamma \sin\theta$.

C5.11: Mathematical Geoscience

Q1: Part (a) was quite standard and was done well by most candidates, although some explanations of stability lacked a good argument. Part (b) was also mostly fine, although a number of people wrote dimensionally inconsistent statements about what they were assuming small in the approximations. Part (c) was easiest done by using the expression for relative humidity derived in part (b), which a few candidates did - it was common to find expressions in terms for ϕ of c without any argument for why c should be constant, and equal to v.

Q2: This was the most popular question, although probably also the most difficult. Part (a) and (b) were quite standard and were successfully completed by the majority of candidates, although a very common issue in part (b) was not understanding (or at least not explaining) that instability would occur if there were *any* k for which the real part of the growth rate were positive. In part (c), besides many algebraic mistakes a common mistake was to include a term H_y in the linearised mass conservation equation. The final part attracted many completely wrong guesses, with very few candidates obtaining the correct answer (some others were close).

Q3: This question was done quite well by some candidates and confused some others who followed a related question in the lectures much too closely without understanding the different boundary conditions considered here. Part (a) was mostly fine. In Part (b), not many people gave a satisfactory answer to what was wrong with the model if q were positive other than that their solution becomes complex at some time. The unseen material in part (c) was tackled well by quite a few candidates, with others being completely confused.

C5.12: Mathematical Physiology

The five candidates split their choice of the three questions evenly. Question 1 was done well which was encouraging, as the last part was quite novel. Question 2 was less well done; the physiologically heavy early part was treated well, but the analytic last part required an understanding of approximation methods for ordinary differential equations which escaped the three attempts. Question 3 was comfortably done.

C6.1: Numerical Linear Algebra

This was generally a successful exam with a range of marks including many high ones. There were a surprisingly large number of candidates who made significant attempts at all three questions rather than concentrating on two as required. Possibly because of the open book format, irrelevant bookwork material that was not asked was described by some.

The first question on the Singular Value Decomposition was attempted by almost all candidates and was generally done reasonably well. Some candidates were a little sloppy with their arguments, in particular in part (c) even if they identified a correct Polar Decomposition which some did not. A surprisingly large number believed that the sum of the singular values was equal to the matrix trace.

The second question on Chebyshev polynomial iterative methods was also popular and reasonably well done. The final part was again found challenging by many who either did not attempt it or, more usually, did not make any headway with it.

The third question on GMRES was attempted by just under half of the candidates and also attracted a range of marks: most attempts seem to have been purposeful and not just rushed in the last few minutes of the exam. The first four parts were generally well done, but the final two unseen parts either attracted full or zero marks in general. There were very few clean answers to the final part.

C6.2: Continuous Optimisation

All questions were attempted by various students. The performance was particularly good especially on Problem 2. Problem 1 was also very accessible, with a trickier bit in part b) and incomplete answers in part c). Problem 3 was not difficult and the students did well on part a), apart from some incomplete answers arguing uniqueness or existence. The theoretical part in Problem 3 was generally incomplete, probably due to time constraints.

C6.3 Approximation of Functions

Problem 1. As often happens all students who took the exam did the first problem, on Chebyshev expansions. Most got marks in the range 23-25, showing good ability with the material and also good ability to in some cases to exploit related material in the textbook in this open book context.

Problem 2. Not surprisingly the final 7-mark part of the problem, on confluent interpolation, proved the most difficult.

Problem 3. The last part, worth 10 marks, was the most challenging, and not many students managed a full solution.

Overall I think this was a successful examination under unfamiliar conditions.

C6.4: Finite Element Methods for Partial Differential Equations

Q1: This question was attempted by 38% of candidates. It revealed a good spread of abilities across those who attempted it. Q1 (b) was for the most part well answered, although some candidates neglected to consider the normal component of the gradient so as to apply the second factorisation lemma. Q1 (c) was very well answered by all who attempted it, although some candidates neglected to provide a counterexample (i.e. computed the determinant of the matrix to be zero, without exhibiting an element of the kernel). Q1 (d) was quite challenging, with no student successfully completing Q1 (d) (ii).

Q2: This question was very popular, with every candidate attempting it. Unsurprisingly, most candidates did very well in the bookwork in Q2 (a). Q2 (b) considered the vector-valued equations of linear elasticity; this was handled much better than vector-valued equations in previous examinations, and almost every student successfully integrated the grad-div term by parts, unlike in previous years. In Q2 (b) (ii), several students applied the fact about the Frobenius inner product *twice*, to replace the inner product of the symmetric gradients with the inner product of the gradients to acquire the familiar bilinear form arising in the vector Laplacian. The second application of this was erroneous. In Q2 (b) (iii), surprisingly few students used the hint to derive Young's inequality, and many failed to prove the required bound on the divergence. Q2 (c) was unseen but was generally answered well, indicating a good understanding of Céa's Lemma. Q2 (d) was also unseen, but related to the Stokes equations studied in lectures. It was quite poorly answered. A remarkable number of candidates wrote down only one equation for two unknowns; others neglected the requirement that the bilinear form be symmetric; and no candidate correctly identified the space for the Lagrange multiplier as $L^2(\Omega)$ instead of $L^2_0(\Omega)$ (which is only appropriate in the incompressible limit).

Q3: This question was attempted by 62% of candidates. The early parts of Q3 (a) were answered well, but many students struggled to correctly state the Newton–Kantorovich iteration for systems of partial differential equations in Q3 (a) (iv). Q3 (b) (i) and (ii) were bookwork and were well-answered, but Q3 (b) (iii) challenged those with a weaker grasp of the inf-sup condition. Q3 (c) was a challenging question about refining the bookwork error estimate for noncoercive problems and gave an opportunity for the best candidates to distinguish themselves.

C7.4: Introduction to Quantum Information

Question 1

Students did very well on it and received a high average mark. Some struggled with part (e) and confused it with Simon's problem. Single marks were also lost in parts (a) e.g. some students viewed the Hadamard transformation as a preparation of an equal superposition of all states in the Hilbert space or of entangled states and part (b) when no justification to the minimum number of calls was provided.

Question 2

This was the most popular question. Students knew their bookwork pretty well. Most of them found part (f) difficult for it required thinking and explaining rather than manipulating equations.

Question 3

This was the least popular question. The bookwork parts (a-c) presented no difficulties and students did very well on them. The most challenging were parts (d), (e) and to some extend (f). In part (d) some students struggled with the probabilistic nature of the algorithm and the estimates of the success probability. In part (e) only few managed to use the spectral decomposition to derive the required formula in few basic steps. Some students, managed part (f) without completing part (e).

C7.5: General Relativity I

Question 1

This question was the most popular of the three questions, and was generally done well by a lot of students. The bookwork parts ((a) and (b)) were almost exclusively done correctly. The new parts ((c) and (d)) caused some more problems; particularly part (d). Here, the most common issue was that students were able to correctly write down expressions for the motion of the second light signal in the form of integrals, but did not notice that, since they were only asked to produce an answer correct to $O(\Delta\tau)$, they could differentiate these expressions in $\Delta\tau$ to obtain the result. Another common mistake was, when considering the motion of the satellite, to consider its radial position as a function of its proper time (that is, $r(\tau_S)$) but to neglect the dependence of the time coordinate t on the proper time τ_S. This is a conceptual issue that is most likely caused by students familiarity with flat space, where one doesn't need to consider such things.

Question 2

This question was by far the least popular, most likely as it appears the most technical and mathematical of the three options. However, it was generally completed very successfully by the small number of students who attempted it. This question did include a typo: in part (c), the order of the arguments of λ should have been reversed, writing $\lambda_{[a\mu+\mu',X]}$ instead of $\lambda_{[X,a\mu+\mu']}$. Fortunately this did not appear to cause any confusion (although it was commented upon by some students!), as the rest of the notation was consistent. Where mistakes were made in this question, the typical error was to believe that the commutator is C^∞ linear (so $[X, aY] = a[X, Y]$ for vectors X, Y and a scalar field a, which is not true) and then to compensate for this error by also failing to properly apply the Leibniz rule for vector fields.

Question 3

Part (a) was done very successfully, with students demonstrating knowledge of the Einstein equations and the various terms appearing in this equation. Part (b) was also bookwork, and was done correctly by many, although a fair number of students were confused regarding the difference between proper time and a general affine parameter. Part (c) was done very successfully, and part (d) caused by far the most problems for students. Here a common mistake was to assume that a geodesic which is initially radial will remain so – this true for the spacetime in question (which is spherically symmetric), but required some justification (it is not true, for example, in the Kerr spacetime). Some students attempted to solve the geodesic equations directly instead of making use of conserved quantities, and this inevitably led to difficulties. A few students also struggled with the integral that needed to be done in part (d) (i), obtaining logarithmic expressions instead of trigonometric ones.

Summary

Overall the quality of the answers was very high. The external examiners' considered this year's exam to be difficult, and yet the students scored very well – I am impressed, especially given the difficult circumstances this year. The spread of questions attempted was as expected, and (also as expected) the more mathematically minded students who attempted question 2 typically did very well. In general, *FHS* students did slightly better than *MSc* students.

C7.6: General Relativity II

- Question 1: This question was attempted by all students. Part a) was solved correctly by the majority of students, as was part b) i), although a few students did not notice here that the result from a) iii) could be used. Part b) ii) was clearly the most difficult question, very few students scored full marks here. Part b) iii) was again easier and nearly everyone scored at least some points here, but at the same time nearly everyone struggled with finding a spacelike geodesic.

- Question 2: This question was the least popular, it was attempted only by a handful of students. Part a) was generally carried out very well, but several students struggled with the concept of gauge in general relativity in part b). Part c) was again correctly solved by almost all students who attempted it. The last part of question 2 was the most difficult one and here only a small number of students presented a good solution.

- Question 3: This question was attempted by nearly everyone. Drawing the Penrose diagrams did not pose any difficulties for the majority of students, however a few students struggled with defining the concept of a black hole. Part b) was also executed well, but a few students did not show that the integral curves of the normal vector field to a null hypersurface are null geodesics. Nearly every student had the right idea for solving c) i), although there were a few computational errors. The determination of the causal character of the hypersurfaces $\{r = const\}$ was again done correctly by nearly everyone, but many students struggled with the computation of the surface gravity. Most students gained some points for c) iii) & iv), but hardly anyone gave full solutions here.

C8.1: Stochastic Differential Equations

Question 1: This was the most popular question. The majority of the students completed parts (a)-(c) without any significant problems. A few students failed to provide a counter-example to Levy's theorem for semi-martingales in part (d). In part (e), most students correctly computed the quadratic variation process and applied the Burkholder-Davis-Gundy inequality in part (i). However, in part (ii), most solutions proved only that $|X_t|/t^{\alpha}$ converges to zero in expectation as $t \to \infty$, and therefore failed to prove almost sure convergence. Part (f) is a generalization of the Liouville theorem for bounded harmonic functions on the whole space. Many students had trouble decomposing the SDE into its bounded variation and martingale parts in order to prove the convergence by appealing to part (i). In part (ii) most students correctly applied Itô's formula to deduce that $f(X_t^x)$ is a continuous local martingale, but only a small number correctly used the boundedness of f, the martingale convergence theorem, and part (i) to deduce that f must be constant.

Question 2: This was by far the least popular question. Parts (a) and (b) did not present any significant problems. Many complete solutions were submitted for part (c) but most repeated ideas of the proof from class to derive the equation for $X_t^1 \wedge X_t^2$, and then appealed to uniqueness in law, as opposed to applying the Tanaka-Meyer formula to $|X_t^1 - X_t^2|$ directly. Part (e) was the most difficult part of the problem, with only a few students achieving totally correct solutions. Most students correctly deduced that if $a < 0$ then the local time is zero owing to the fact that the support of the measure defined by the local time is contained in the set $\{|X_t| = a\}$. The correct solutions then essentially separated the event $\{|X_t| = a\} = \{X_t = a\} \cup \{X_t = (-a)\}$ and used the definition of the local time to complete the proof. In part (f) most students correctly applied the Dambis-Dubins-Schwarz theorem and then justified the equality of local times using the definition.

Question 3: This was the second most popular question, falling only slightly behind Question 1. Recalling properties of the stochastic exponential in part (a) did not present any significant problems. Most students correctly used the stochastic exponential to solve the SDE in part (b), the integration by parts formula to prove uniqueness, and the dominated convergence theorem to prove the convergence as $\varepsilon \to 0$. Part (c) was an application of a random time-change and the Girsanov formula. Proofs of these facts essentially appear in the course notes, and the majority of students provided essentially complete solutions. Only a few students achieved complete solutions to part (d). In part (i), the students were asked to bound separately the the bounded variation and martingale parts of the solution, using basic integration theory to bound the first and the Burkholder-Davis-Gundy inequality to bound the second. Very few students had correct solutions to part (ii), which recalled aspects of the uniqueness proof for solutions to SDEs with Lipschitz continuous coefficients, and required the use of Grönwall's inequality.

C8.2: Stochastic Analysis and PDEs

Question 1

The bookwork part of the question (parts (a), (b)(i), and (b)(iii)) caused no problems. Showing that A is a Markov pregenerator in (b)(ii) was also done generally well, though a common error was that candidates failed to consider the case when the minimum of a function $f \in C^2(\mathbb{R})$ on $[-1, 1]$ occurs at a boundary point $x \in \{-1, 1\}$, in which case one does not necessarily have $f'(x) = 0$ and $f''(x) \geq 0$. Part (b)(iv) caused some trouble, with no candidate earning full marks for this subpart. Many candidates argued correctly, using the Hille–Yosida theorem and related results, that it suffices to show that the range of $I - \lambda A$ is dense for all $\lambda \geq 0$, and then attempted to show that the polynomials are in the range of $I - \lambda A$. This reduces the problem to a linear system of equations, but none argued correctly why this system is solvable. Part (c) proved to be difficult, with few candidates making even partial progress. However several did realise that the core of the problem was to show that $(\lambda - A)(\lambda_n - A)^{-1} \to I$ in an appropriate sense.

Question 2

All candidates attempting this question answered parts (a)(i) and (a)(ii) correctly, but some made errors in (a)(iii) by failing to state that the convergence of coefficients in the Stroock–Varadhan theorem has to happen locally uniformly rather than only pointwise. Part (b) was done well, with calculations in parts (i)-(ii) of the scale function, density of the speed measure, and Green's function causing no problems. Only (b)(iii) caused minor problems where some candidates could not finish the calculation or did not justify the divergence of the integral. Part (c) proved to be the most troublesome, with some candidates using an incorrect scaling in (c)(i)-(ii), though most did realise that part (a)(ii) should be used in (c)(i) to show that the limiting process is deterministic. Part (c)(iii) was not done correctly by any candidate, the most common error being that candidates stated (correctly) that the conditions of the convergence theorem in (a)(iii) are not satisfied for any other choice of α, but did not actually argue that convergence does not occur.

Question 3

This was the most popular question with every candidate attempting it. Parts (a) and (c)(i), which were standard bookwork, caused almost no problems, with only (a)(iii) confusing some candidates about how they should add together the solutions to the two PDEs. Part (b) was a variation of material covered in lectures and of a previous exam question. Many candidates answered (b) correctly and most made partial progress. Many did not realise, however, that the assumption that u is C^2 on $\mathbb{R} \times \mathbb{R}^d$, rather than only on $[0, T] \times U$, allows one to avoid the general time-change argument seen in lectures and thus simplify the proof. In part (c)(ii), all candidates showed that Brownian motion in $\mathbb{R}^2$ eventually hits any fixed ε-ball $B(x, \varepsilon)$, however few were able to completely argue why this implied that Brownian motion is neighbourhood recurrent. Most candidates realised that they should use the strong Markov property, and many argued that there exists a sequence of stopping times $\tau_1 \leq \tau_2 \leq \ldots$ such that $B_{\tau_k} \in B(x, \varepsilon)$, but failed to argue that $\lim_{k\to\infty} \tau_k = \infty$ which is necessary for neighbourhood recurrence (in fact, several constructions yielded $\tau_1 = \tau_2 = \ldots$ almost surely, for which this property does not hold). Several candidates argued well how to pass from almost surely hitting a given ε-ball infinitely often to almost surely hitting every open set infinitely often, for which they used balls $B(x, \varepsilon)$ with rational x and ε as seen in the lectures. Most candidates made partial progress on the final part (c)(iii) and several gave complete answers. The most common error was to assume that the condition $\sup_{x \in \mathbb{R}^2} u(x) < \infty$ implies that the local martingale $u(B_t)$ is bounded.

C8.3: Combinatorics

1. Question 1 was attempted by the vast majority of candidates. Many attempts lacked precision in part (a)(i), with most candidates forgetting to provide an extremal example of an intersecting family. Part (a)(ii) required a generalisation of the usual argument for intersecting families, and this was very well attempted given that it was unseen. Part (a)(iii) was a minor modification of a question from example sheet 4, and most candidates did well with this.

   The purpose of the bookwork in part (b) was to get candidates thinking along the right lines for part (c) – all students got these marks. There were then surprisingly few successful attempts at (c)(i), given that the dot products $v_i \cdot v_i$ and $v_i \cdot v_j$ are obtained from the intersection sizes in a standard way, and the hint provides the extra idea needed. Part (c)(ii) was much more difficult and, as expected, completed only by the top candidates.

2. Question 2 was extremely unpopular, perhaps because the Sauer-Shelah lemma does not feel like a central part of the course. The bookwork in part (a) should have reminded candidates of the statement of the Sauer-Shelah lemma and prepared them for (b)(iii). All attempts at the question obtained these bookwork marks. A number of candidates then demonstrated sufficient grasp of the (complicated) definition of T_r to be able to answer (b)(i) and (b)(ii). Virtually no marks were obtained in (b)(iii) which was disappointing – the proof differs very little from the proof of the Sauer-Shelah lemma. Indeed much of part (a) can be re-used here, if we replace the words 'shatters a k-set' with the words 'contains a T_k'. Very few candidates had a serious go at the last part, but in fact they had the right key idea.

3. Question 3 was the most popular question on the paper, perhaps because the part of the course covering antichains and LYM is felt to be quite accessible. The question also had the highest average mark, so in general people found the strong antichain more easy to get to grips with than the definitions in the other questions.

   The bookwork in parts (a) and (b) was intended to remind students of LYM and the ideas of compression which can be helpful in part (c). These marks were almost all awarded, with the exception of a few candidates who failed to provide the equality cases. Part (c)(i) was tackled very well by the majority of candidates, with several finding slight variants of the expected approach. There was an impressive level of engagement with this part, and the success rate was high. Part (c)(ii) was also fairly well-tackled, but people found (c)(iii) much harder (considering $B_1 \cap B_2$ rather than $B_1 \setminus B_2$ was generally the key mistake). There were lots of attempts at (c)(iv), but this was a trickier test of the student's understanding of good/bad sets and only about half of the answers had the right main ideas.

Summary The exam created a reasonable spread of marks, with a few candidates managing full marks. The average marks for Q1 were a bit lower than for Q3. Q2 was too unpopular to generate meaningful data – the sample size was too small. This was probably due to a combination of factors: the definition of T_r was perhaps more impenetrable than the other definitions, and it may have appeared that there was less bookwork in Q2 than in the other questions (this is technically true, although Q2(b)(iii) is a very minor modification of the bookwork). Given the open book format, it is unsurprising that almost every bookwork mark was awarded, but these parts still played an important role in setting the scene for each question.

C8.4: Probabilistic Combinatorics

Question 1 was most popular; with hindsight it was a bit too straightforward for an open book exam, though it did still distinguish candidates to some extent. Almost (but not quite) everyone managed (a). The counting is slightly tricky to get right in (b)(i) for $k = 6$; some scripts were incorrect or unclear. For $k = 5$ it's enough just to say that the host graph is bipartite and contains no odd cycles. For the main part, (b)(ii), the lengths of the answers varied very considerably. Some explanation is needed of the various cases involved in the variance estimate, but this can be complete, clear and concise. (Of course, complete, clear long answers also obtained full marks.) For (b)(iii) the example requested has to be in the random graph model in the question, not in $G(n, p)$ – no credit was given for a $G(n, p)$ example, which is in the lecture notes. Not so many candidates managed this part, which is disappointing.

Question 2 distinguished quite well. Many candidates could not describe the condition for equality in Harris's Lemma, although it was on a problem sheet. In part (b) some translation from the set context to the graph context was expected, along with defining the notion of an increasing property of graphs (on a given vertex set). Parts (c) and (d) were mostly fairly well done, though in (d) at least some brief explanation of why Harris's Lemma applies was expected and not always present.

Question 3 turned out to be harder than intended, and was marked correspondingly generously. In (b)(i) it is quite hard to obtain the bound with the given constant (involving a non-standard way of generating the dependency digraph). A few candidates did manage this, and more thought they had with an incorrect argument. Very significant credit was given for a bound with a worse constant. (ii) is very similar to (i) (just indicating what changes is enough). (iii) was again quite hard, though the basic idea - apply the general form of the Local Lemma with two kinds of events - has been illustrated in lectures, and the hint should help with the needed calculations. Not many candidates attempted (iv), though it has a very simple solution (e.g., many disjoint copies of P_4).

C8.5: Introduction to Schramm-Loewner Evolution

Question 1 Parts (a) and (b) are standard bookwork which was well done. Part (c)(i) is also bookwork. It could be done in many ways, in particular by the conformal invariance of the harmonic measure. In part (c)(ii) the crucial observation is that any BM hitting $K \cup \mathbb{R}$ must hit $\{|z| = 1, \Im z \geq 0\} \cup \mathbb{R}$ first. The only non-trivial contribution to the expectation of $\Im B_\tau$ comes from trajectories that hit the upper half-circle. Part (c)(iii) follows from the formula in part (ii) and an estimate of the probability that a Brownian motion will hit the 'top side' of K.

Question 2 Parts (a) and (b) are standard bookwork. Part (c) boils down to the question whether $B_{t+s} - B_s$ has the law of the Brownian motion. There are many examples showing that this might not be the case is s is not a stopping time. Part (d) is a bit trickier than it seems. There are two main issues: (1) in order to claim that $g_s(\gamma) - u_s$ is SLE one need to know that s is a stopping time (2) although g_t is a.s. continuous up to the boundary for fixed t, it is not obvious that this is true for all values of t. This part was no done well. Part (e) was not attempted by any candidate.

Question 3 This is by far the easiest question for an open book exam. Most of it is essentially the same as a computation from the lectures. It is a classical application of the 'martingale trick' and the optional stopping theorem in the SLE context. This question was rather well done by all candidates.

C8.6: Limit Theorems and Large Deviations in Probability

All candidates attempted question 1 and question 2. Question 1. All candidates provided excellent solutions. Book work are done with precise and complete answers. Part (b(ii)) and Part (c) are new but similar questions are seen in classes. While candidates' solutions are excellent and contain all the details more than required. Question 2. All candidates did quite well on Part (a) of the question, which is mainly book work or similar seen in their classes. Part (b) proved challenging and new, while candidates understood well the approach they should use, but failed to identify the continuity mapping explicitly.

Statistics Units

Reports on the following courses may be found in the Mathematics and Statistics examiners' report.

SC1 - Stochastic Models in Mathematical Genetics SC2 - Probability and Statistics for Network Analysis SC4 - Advanced Topics in Statistical Machine Learning SC5 - Advanced Simulation Methods SC7 - Bayes Methods SC9 - Interacting Particle Systems SC10- Algorithmic Foundations of Learning

Computer Science

Reports on the following courses may be found in the Mathematics and Computer Science examiners' report.

Quantum Computer Science Categories, Proofs and Processes Computer Animation

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